February 1, 2008

NONCOMMUTATIVE ELECTRODYNAMICS AND ULTRA HIGH ENERGY GAMMA RAYS

Paolo Castorina[1,2], Dario Zappalà[2,1]

[1] *Dipartimento di Fisica, Università di Catania*
Via S. Sofia 64, I-95123, Catania, Italy

[2] *INFN, Sezione di Catania, Via S. Sofia 64, I-95123, Catania, Italy*

e-mail: paolo.castorina@ct.infn.it; dario.zappala@ct.infn.it

Abstract

Plane waves in noncommutative classical electrodynamics (NCED) have a peculiar dispersion relation. We investigate the kinematical conditions on this deformed "mass shell" which come from ultra high energy gamma rays and discuss noncommutative dynamical effects on the gamma absorption by the infrared background and on the intrinsic spectrum. Finally we note that in NCED there is a strong correlation between the modified dispersion relation and the presence of dynamical effects in electromagnetic phenomena such as in the case of the synchrotron radiation. From this point of view, the limits on the typical energy scale of the violation of Lorentz invariance obtained by deformed dispersion relations and by assuming undeformed dynamical effects should be taken with some caution.

PACS numbers: 11.10.Nx 95.85.Pw 96.40-z

Recently there has been a growing interest in theories where the speed of light is different from c [1]. The main motivations of considering this 'heresy' come from cosmology, quantum gravity and observation and indeed, ultra high energy cosmic (UHECR) and gamma (UHEGR) rays have been detected with energies which seem to be inconsistent with the standard GZK cutoff [2].

It has been suggested that a possible explanation of the observed high energy cosmic rays could be a modified dispersion relation among energy, momentum and mass due to physical phenomena at Plank scale [3, 4, 5, 6, 7, 8] or to explicit Lorentz invariant breaking terms in the lagrangian, originally proposed in [9] and reconsidered more recently in [10].

Also in noncommutative electrodynamics (NCED) [11] one has a modified dispersion relation for the electromagnetic waves and in this brief note we address the question whether it is compatible with the experimental data on UHEGR and we discuss some noncommutative dynamical effects in gamma rays absorption by the diffuse interstellar or intergalactic infrared radiation and in the TeV photon intrinsic production spectrum.

The simplest case of NCED is described by the action (the fermionic field is omitted):

$$\hat{I} = -\frac{1}{4}\int d^4x \, [F^{\mu\nu}F_{\mu\nu} - \frac{1}{2}\theta^{\alpha\beta}F_{\alpha\beta}F^{\mu\nu}F_{\mu\nu} + 2\theta^{\alpha\beta}F_{\alpha\mu}F_{\beta\nu}F^{\mu\nu}] \,, \tag{1}$$

where the noncommutativity of space-time coordinates has been expressed in the canonical form [12], by

$$x^\mu * x^\nu - x^\nu * x^\mu = i\theta^{\mu\nu} \,, \tag{2}$$

the $*$-product is the standard Moyal product ([13]); $\hat{F}_{\mu\nu} = \partial_\mu \hat{A}_\nu - \partial_\nu \hat{A}_\mu - i[\hat{A}_\mu, \hat{A}_\nu]_*$ and $\hat{A}_\mu$ has been expressed in terms of a U(1) gauge field A_μ by the $O(\theta)$ Seiberg-Witten [14] map

$$\hat{A}_\mu(A,\theta) = A_\mu - \frac{1}{2}\theta^{\alpha\beta}A_\alpha(\partial_\beta A_\mu + F_{\beta\mu}) \,. \tag{3}$$

In [11] it was found that, in the presence of a background magnetic field $\vec{b}$, the $O(\theta)$ plane-wave classical solutions exist. The waves propagating transversely to $\vec{b}$ have a modified dispersion relation

$$\omega/c = k(1 - \vec{\theta}_T \cdot \vec{b}_T) \tag{4}$$

(where $\vec{\theta} \equiv (\theta^1, \theta^2, \theta^3)$, with $\theta^{ij} = \epsilon^{ijk}\theta_k$, and $\theta^{0i} = 0$) while the ones propagating along the direction of $\vec{b}$ still travel at the usual speed of light c.

For completeness, let us remark that the quantum theory of noncommutative electrodynamics is still not completely understood. The perturbative calculations show a novel feature of the theory, called infrared-ultraviolet (IR-UV) connection[13], that prevents to take the limit of the noncommutative parameter to zero and, moreover, the photon self energy introduces, at one loop order in perturbation theory, a tachyonic pole which goes to minus infinity. However, in our opinion the IR-UV connection could be for instance an effect induced by the application of standard perturbative techniques or by a restricted theoretical framework. For example in [15] (see also [16]) it is shown that there are (scalar) field theories where the IR-UV connection is absent and its presence depends on the projection on the noncommutative plane. Due the present status of the quantum formulation of the noncommutative theory, we assume the conservative point of view of working in the framework of the classical noncommutative theory which, for $\vec{\theta} \to 0$, reproduces the standard electrodynamics.

The new dispersion relation in Eq. (4) has many physical consequences which are however not easily observable. As a matter of fact, by using the bound of $\theta < 10^{-2}(TeV)^{-2}$ [17], one would need a background magnetic field of the order of 1 Tesla over a distance of 1 parsec to appreciate the shift of the interference fringes due to the modified speed of noncommutative light. Mainly for this reason, other classical and quantum phenomena have been recently proposed to improve the bound and to find new applications of noncommutativity [18].

It is then interesting to verify whether the deformed dispersion relation in Eq.(4) is compatible with the astrophysical observations of UHEGR [19] [20] and, more generally, what kind of phenomenological limits on the energy scale which characterizes the violation of the Lorentz invariance are obtained by a dynamical, and not purely kinematical, analysis of the NCED effects.

Let us start by evaluating the effects of the dispersion relation in Eq.(4) on the kinematical threshold for $\gamma \to e^+e^-$ and $\gamma + \gamma \to e^+e^-$.

In Eq.(4) the noncommutative contribution depends on the angle between the transverse components (with respect to $\vec{k}$) of the background magnetic field $\vec{b}$ and of the vector $\vec{\theta}$. At the first order in θ one has

$$\omega(1 + \vec{\theta}_T \cdot \vec{b}_T)/c = k \tag{5}$$

and the "mass shell" relation becomes

$$E_\gamma^2 - k^2 = -2E_\gamma^2(\vec{\theta}_T \cdot \vec{b}_T). \tag{6}$$

where $E_\gamma = \omega/c$.

In the case of NCED, with no modified dispersion relation for the electron, the decay $\gamma \to e^+e^-$ is kinematically permitted [10]. On the other hand this decay is forbidden, for example, with the dispersion relation

$$E^2 - k^2 = -E^3/E_{QG} \tag{7}$$

which comes from quantum gravity effect with typical energy scale E_{QG} [3]. More general relations of the form in Eq. (7) which include phenomenological parameters, have been considered in [4, 6].

By defining p_γ the four momentum of the photon and p_+ and p_- the four momenta of e^+ and e^-, the kinematical condition for the decay is

$$-2E_\gamma^2(\vec{\theta}_T \cdot \vec{b}_T) = (p_+ + p_-)^2 > 4m_e^2 \tag{8}$$

which requires $(\vec{\theta}_T \cdot \vec{b}_T) < 0$. Since gamma rays with energy $\simeq 50\ TeV$ have been observed [19] from the Crab Nebula this implies that

$$(\vec{\theta}_T \cdot \vec{b}_T) > -2 * 10^{-16} \tag{9}$$

The differences between the dispersion relation for NCED and Eq. (7) are that in our case the parameter which modifies the relation can be negative, that is the speed of noncommutative light is less than c, and that there is a different dependence of the modification on the energy (quadratic vs. cubic).

Let us notice that our previous result is essentially the same kinematical limit obtained in [10] where however the physical mechanism is the difference in the maximum speed between the electron and the

photon while in our case the mass shell relation for the electron is unchanged. Moreover in NCED the modification depends on an external parameter, the background magnetic field. Indeed, in the limits on $(\vec{\theta}_T \cdot \vec{b}_T)$ one has to specify if $\vec{b}$ is , for example, the galactic or extragalactic magnetic field. In discussing the limit from the observation of gamma rays from the Crab nebula $\vec{b}$ is the galactic magnetic field which is of order $b_g \simeq 1\mu G$.

Let us now consider the limit which comes from $\gamma + \gamma_b \rightarrow e^+e^-$ where γ_b is a background low energy photon with momentum $p_b = (E_b, \vec{p}_b)$. In this case the kinematical condition is

$$-2E_\gamma^2(\vec{\theta}_T \cdot \vec{b}_T) + 2E_bE_\gamma - 2E_bE_\gamma \cos\phi(1 + \vec{\theta}_T \cdot \vec{b}_T) > 4m_e^2 \tag{10}$$

where the effect of the modified dispersion relation has been neglected for the low energy photon (because numerically irrelevant) and ϕ is the angle between the momenta of the photons. Since the observation of extragalactic gamma rays up to energy $\simeq 20\ TeV$ is in agreement with the absorption by the infrared diffuse extragalactic background [21], let us consider the energy E_b is in the range $1\ meV - 200\ meV$ to obtain the kinematical constraints on the noncommutative parameter. Then, for a central collision $(\cos\phi = -1)$, the condition is

$$-(\vec{\theta}_T \cdot \vec{b}_T) > +2(m_e/E_\gamma)^2 - 2(E_b/E_\gamma) \tag{11}$$

which for $E_b = 1\ meV$ gives a negative value

$$(\vec{\theta}_T \cdot \vec{b}_T) < -1.15 * 10^{-15} \tag{12}$$

while for $E_b = 200\ meV$ gives

$$(\vec{\theta}_T \cdot \vec{b}_T) < 1.9 * 10^{-14} \tag{13}$$

In combining the kinematical conditions in Eqs. (9), (12) and (13) we have to take into account that the extragalactic background magnetic field b_{eg} is about $10^{-3}\ b_g$. Then, for a far infrared extragalactic absorption ($E_b = 1\ meV$) the conditions, Eq. (9) and Eq. (12), cannot be simultaneously satisfied. Indeed, the combined constraints limit the range of E_b for absorption of UHEGR. If one evaluates Eq. (11) with the value in the right hand side of Eq. (9), it can be easily seen that the production $\gamma + \gamma_b \rightarrow e^+e^-$ is possible only for energy of the background photon $E_b > O(10\ meV)$. This value of E_b is close to that one which maximizes the standard electrodynamics pair production absorption cross section [22].

In the second case ($E_b = 200\ meV$) the conditions are

$$-2 * 10^{-16} < \vec{\theta}_T \cdot \vec{b}_{gT} < 1.9 * 10^{-11} \tag{14}$$

If we consider the limit $|\theta| < (10\ TeV)^{-2}$ and the magnitude of the galactic or extragalactic magnetic field, the previous conditions are satisfied by many order of magnitudes:

$$|\vec{\theta}_T \cdot \vec{b}_{gT}| < |\theta_T||\vec{b}_{gT}| \simeq 10^{-30} \tag{15}$$

Then the dispersion relation coming from NCED is largely consistent with the present data on UHEGR. The previous results could be an effect of the simplified version of the theory here considered. Indeed, in the complete version of NCED also a modified dispersion relation for electrons and

positrons could possibly be present. However we do not believe that this effects would dramatically change the previous conclusions and moreover the problem of mass generation in noncommutative field theory is still open [23] as well as the possibility to have a full consistent quantum field theory [24].

In addition to the kinematical constraints discussed above, there are many other noncommutative dynamical effects for example in the absorption cross section and in the source gamma ray spectrum. In fact the standard calculations to fit the observed gamma ray data [22] require the evaluation of the optical depth for attenuation between the source and the Earth due to the $\gamma + \gamma_b \to e^+e^-$ cross section and the most widely investigated model for the production of TeV photons involves the injection of relativistic electrons via the synchrotron self-Compton mechanism [25].

While from the perturbative quantum noncommutative calculations, where the modifications in the dispersion relations are neglected, a very small effect to the $\gamma + \gamma_b \to e^+e^-$ process is predicted [26] in the energy range $E_b \sim 100\ meV$, $E_\gamma \sim 20\ TeV$ with $|\theta| < (10\ TeV)^{-2}$, it has been shown in [27] that a consequence of the modified dispersion relation, as in Eq. (4), is a large correction to the *classical* synchrotron spectrum of a relativistic particle in a strong magnetic field. As an example, the correction to the synchrotron spectrum for a relativistic electron to $O(\theta)$, for $|\theta| < (10\ TeV)^{-2}$ and for $\omega_0 << \omega << \omega_0\gamma^3$ turns out [27]

$$X = \frac{dI(\omega)/d\Omega}{dI(\omega)/d\Omega|_{\theta=0}} < 1 + \left(\frac{\omega_0}{\omega}\right)^{2/3} n \times 10^{-21} \times (E_{electron}(MeV)/(MeV))^4\ , \tag{16}$$

where ω_0 is the synchrotron frequency, ω is the radiation frequency, γ is the Lorentz factor, $E_{electron}$ is the energy of the electron and n is the value of the magnetic field, which accelerates the electron, expressed in Tesla. If a 20 TeV photon is produced by synchrotron radiation, the correction is at least

$$X < 1 + 1.6\left(\frac{\omega_0}{\omega}\right)^{2/3} n \times 10^8, \tag{17}$$

The absolute value of the correction depends on the frequencies and on the magnetic field one is considering, but this result is a signature that the modification of the dispersion relation is strongly correlated with dynamical effects. From this point of view, the limits on the typical energy scale of the violation of Lorentz invariance obtained by deformed dispersion relations and by assuming undeformed dynamical effects should be taken with some caution.

In conclusion our analysis shows that the kinematical constraints form the UHEGR are too weak to give meaningful indications on the noncommutativity parameter θ. Conversely, we expect that the NCED could play a relevant role in the dynamical processes involved in the intrinsic production spectrum of the UHEGR, as the sychrotron self-Compton mechanism, thus providing a possible mean to put a tight bound on θ.

Acknowledgements

We are grateful to G. Nardulli for many indications and suggestions on the subject and to A. Iorio for stimulating discussions.

References

[1] S. Liberati, S. Sonego and M. Visser, Annals Phys. **298** (2002) 167; J. Magueijo, "New varying speed of light theories", arXiv:astro-ph/0305457.

[2] K. Greisen, Phys. Rev. Lett. **16**, (1966) 748; G. T. Zatsepin and V.A. Kuzmin, Sov. Phys. JETP Lett. **4**, (1966) 78.

[3] G. Amelino-Camelia, J.R. Ellis, N.E. Mavromatos, D.V. Nanopoulos and S. Sarkar Nature **393**, (1998) 763; R. Gambini and J. Pullin, Phys. Rev. **D59** (1999) 124021.

[4] G. Amelino-Camelia and T. Piran, "Planck scale deformation of Lorentz symmetry as a solution to the UHECR and the Tev-γ paradoxes", arXiv:astro-ph/0008107.

[5] G. Amelino-Camelia, "Improved limit on quantum space-time modifications of Lorentz symmetry from observations of gamma-ray blazars", arXiv:astro-ph/0212002.

[6] T. Jacobson, S. Liberati, D. Mattingly , "Lorentz violation and Crab synchrotron emission: a new constraint far beyond the Planck scale", arXiv:astro-ph/0212190.

[7] T. Jacobson, S. Liberati, D. Mattingly , "Comment on "Improved limit on quantum space-time modifications of Lorentz symmetry from observations of gamma-ray blazars"", arXiv:gr-qc/0303001.

[8] , F.W. Stecker, "Tests of quantum gravity and extra dimensions models using high energy gamma rays observations", arXiv:astro-ph/0304527.

[9] S.M. Carroll, G. B. Field, R. Jackiw, Phys. Rev. **D 41** (1990) 1231.

[10] S. Coleman and S.L. Glashow, Phys. Rev. **D 59** (1999) 116008; F.W. Stecker and S.L. Glashow, Astrop. Phys. **16** (2001) 97.

[11] Z. Guralnik, R. Jackiw, S.Y. Pi, A.P. Polychronakos, Phys. Lett. **B 517** (2001) 450; R.G. Cai, Phys. Lett. **B 517** (2001) 457.

[12] J. Madore, S. Schraml, P. Schupp, J. Wess, Eur. Phys. J. **C16** (2000) 161.

[13] M.R. Douglas, N.A. Nekrasov, Rev. Mod. Phys. **73** (2001) 977.

[14] N. Seiberg, and E. Witten, JHEP **9909** (1999) 32.

[15] C.-S. Chu, J. Madore, H. Steinacker, JHEP **0108** (2001) 038.

[16] S. Vaidya and B. Ydri, "On the Origin of the UV-IR Mixing in Noncommutative Matrix Geometry", arXiv:hep-th/0305201.

[17] S. Carroll, J. Harvey, V.A. Kostelecky, C. Lane, T. Okamoto, Phys. Rev. Lett. **87** (2001) 141601.

[18] I. Hinchliffe, N. Kersting and Y.L. Ma, "Review of the phenomenology of noncommutative geometry", arXiv:hep-ph/0205040.

[19] Tanimori et al. Astrophys. J. **497** (1998) L25; Astrophys. J. **492** (1998) L33; F.W. Stecker, "Astrophysics at Highest energy frontiers", arXiv:astro-ph/0208507.

[20] F.Krennrich et al., Astrophys. J. **560** (2001) L45; F.A. Aharonian et al., Astron. Astrophys. **393** (2002) 89; F.A. Aharonian et al., Astron. Astrophys. **366** (2001) 62; F.A. Aharonian et al., "TeV gamma-rays from the blazar H 1426+428 and the diffuse extragalactic background radiation", astro-ph/0202072; K. Okumura et al., Astrophys. J. **579** (2002) L9.

[21] A. Konopelko, A. Mastichiadis, J. Kirk, O.C. De Jager, F.W. Stecker, "Modelling the TeV γ-ray spectra of two low redshift AGNs: Mkn501 and Mkn 421", astro-ph/03020049.

[22] F.W. Stecker, O.C. De Jager and M.H. Salamon Astrophys. J. **390** (1992) L49.

[23] S.S. Gubser and S.L. Sondhi , Nucl. Phys. **B581** (2001) 240; P. Castorina and D. Zappalà, Phys. Rev. **D 68** (2003) 065008.

[24] G. Amelino-Camelia, G. Mandanici, K. Yoshida, "On the IR/UV mixing and experimental limits on the parameters of canonical noncommutative spacetimes", arXiv:hep-th/0209254; F. Ruiz Ruiz, Phys. Lett. **B502** (2001) 274; A. Armoni, E. Lopez, Nucl. Phys. **B632** (2002) 240.

[25] O.C. De Jager et al., Astrop. J. **457** (1996) 253.

[26] J. L. Hewett, F.J. Petriello, T.G. Rizzo Phys. Rev. **D 64** (2001) 075012.

[27] P. Castorina, A. Iorio and D. Zappalà , "Noncommutative synchrotron", arXiv:hep-th/0212238.